Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Engineering Division

Presented at the 10th International Magnet Technology Conference, Boston, MA, September 21–25, 1987, and to be published in IEEE Transactions on Magnetics, MAG-24, No. 2

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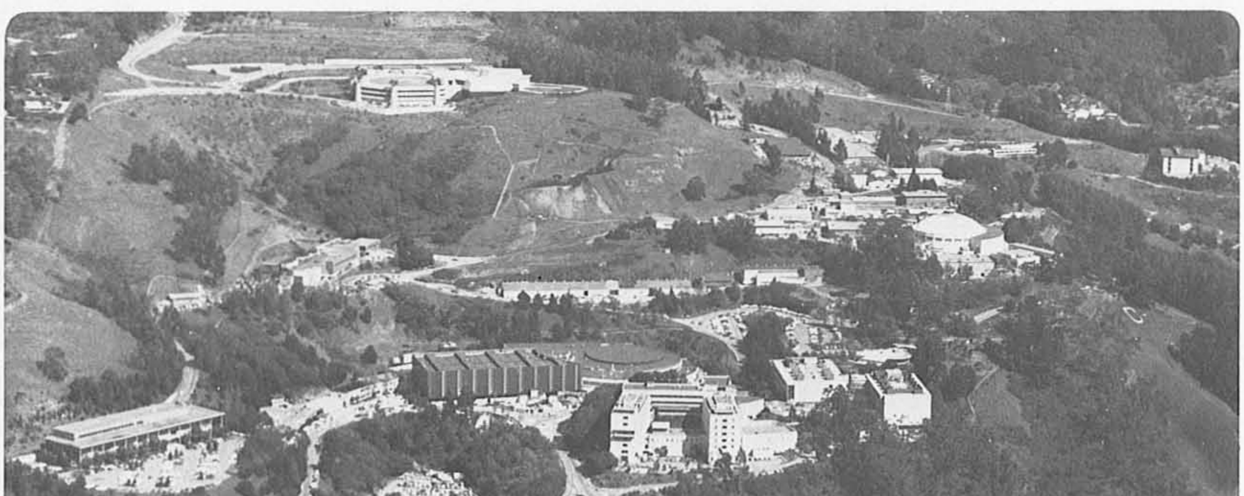
September 1987



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

Modeling the Behavior of Oriented Permanent Magnet Material
Using Current Double Theory

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November 1987

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ABSTRACT

This paper presents a method for modeling two dimensional dipoles, quadrupoles and other higher multipoles built using oriented permanent magnet materials such as samarium cobalt (one of the rare earth cobalt REC materials). The technique presented here uses complex current doublet to model the magnetized material. This technique can be used in conjunction with an infinitely permeable circular iron shield which lies outside the REC material. Examples of two types of dipoles and quadrupoles are presented in this report.

BACKGROUND

Theory for the residual field generated by the circulating currents in the filaments of the superconductor of superconducting dipoles and quadrupoles was published in the early 1970's. [1,2] This theory treats the circulating currents in the superconductor as a current doublet in the complex plane. This current doublet behaves just like a doublet in hydrodynamic theory [3] (a source and a sink together at a single point in the complex plane). The current doublet can be modeled with iron using the method of images.

More recently the SCMAG04 computer program, which uses doublet theory has been applied to dipoles and quadrupole for the SSC. [4,5] The magnetic field generated by circulating currents in the SSC dipoles is rich in higher multipoles. Several methods for bucking out these higher multipoles which use passive methods have been studied. [6,7,8] The magnetization observed in the superconductor can be directly related to the strength of the current doublet. [7,9,10] Thus one finds that the theory for current doublets can be applied to oriented magnetized materials. [7]

Since superconductor magnetization can be related directly to current doublet theory, it follows that permanent magnetization of oriented rare-earth cobalt materials can be modeled using the same theory and the same SCMAG04 computer code provided the magnetic field from currents and other pieces of oriented magnetized materials do not demagnetize the oriented magnetized material in the magnet being studied. [11]

Oriented rare earth-cobalt materials have the interesting property of being transparent to the magnetic field (unlike iron which will concentrate the field into itself). This transparency holds as long as the oriented material is not demagnetized. For example, material like $SmCo_5$ can be magnetized so that the permanent magnetization $\mu_0 M \cong 0.9 - 1.1$ T at room temperature. In order to demagnetize the material, a demagnetizing H of 650 - 750 kAm has to be applied (the demagnetizing induction is 0.82 to 0.95 T) at room temperature. [12] The demagnetizing field is applied in a direction opposite from that of the magnetization. The method of manufacture involves grinding the rare earth cobalt (or related material) into domain sized grains (about 5 μm in diameter). The material is then oriented by vibrating the powder while it is in a strong magnetic field to orient them along the field lines. The material is then bonded together by sintering or some other process to immobilize the oriented grains. The finished blocks can be machined to final dimensions by grinding. [13,14]

MODELING ORIENTED RARE EARTH COBALT MAGNETS USING DOUBLET THEORY

The field given in the complex plane at a location Z by a current I at a location Z_c can be stated using the following expression:

$$H^*(Z) = \frac{I}{2\pi i}\frac{1}{(Z - Z_c)} \tag{1}$$

where $H^*(Z)$ is the complex conjugate of the field H ($H^* = H_x - iH_y$) at the location Z. I is the current at location Z_c in the complex plate. A current doublet is a current pair +I and -I separated by a distance d the strength of the doublet $\Gamma = Id$. The field generated at a location Z by a current doublet with strength Γ at a location Z_c can be stated as follows:

$$H^*(Z) = \frac{\Gamma}{2\pi i}\frac{e^{i\alpha}}{(Z - Z_c)^2} \tag{2}$$

where $H^*(Z)$, Z, Z_c and Γ have been previously defined and α is the angle of the current doublet (see Figure 1). A magnetized superconductor which has a filament diameter of d_f will have a doublet strength factor; [2]

$$\Gamma = \frac{d_f^{\,3}}{6} J_c \tag{2a}$$

when the conductor has been fully penetrated by a field change and the superconductor current density is a uniform value of J_c. The doublet angle α is the angle of the field plus $\pi/2$.

The doublet strength factor Γ can be directly related to magnetization using the following relationship

$$\Gamma = \frac{\pi D^2 M}{4} \tag{2b}$$

where M is the magnetization measured and D is the diameter of the sample. The orientation angle Φ for the rare earth cobalt material can be related to the doublet angle α by the relationship $\Phi = \alpha - \pi/2$ (see Figure 2 and compare it with Figure 1.) Thus one finds that Equation 2 takes the following form for an oriented magnetized material with an orientation of Φ, a diameter D and a magnetization strength of M

$$H^*(Z) = \frac{D^2 M}{8\, i}\frac{e^{i(\Phi+\pi/2)}}{(Z - Z_c)^2} \tag{3}$$

Equation 2 or 3 can be expanded in a Taylor series which takes the following form

$$H^*(Z) = \sum_{N=1}^{\infty} C_N Z^{N-1} \tag{4}$$

Oriented Current Doublet

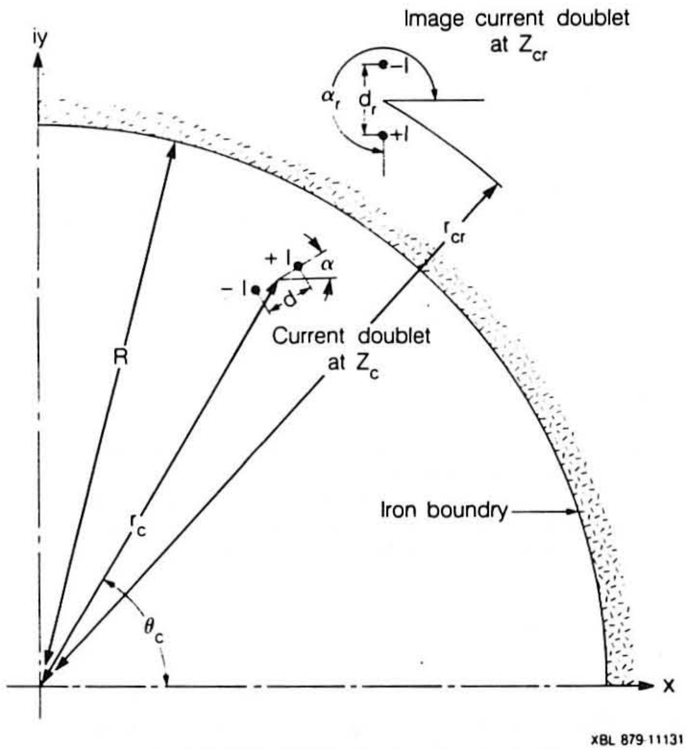

Fig. 1. A current doublet and image current doublet in the complex plane.

where

$$C_N = a_N + b_N \tag{4a}$$

where a_N is the multipole coefficient generated by the doublet itself and b_N is the multipole generated by the doublet in an iron shell. In equation 4, N=1 is the dipole term, N=2 is quadrupole, N=3 is sextupole, and so on.

The a_N term, when applied to the current doublet (Equation 2) takes the following form

$$a_N = - \frac{\Gamma_e^{i\alpha}}{2\pi i} N Z_c^{-(N+1)} \tag{5}$$

where Z_c is the location of the doublet. When a_N term is derived for a small piece of oriented rare earth cobalt the term is

$$a_N = - \frac{D^2 M e^{i(\Phi+\pi/2)}}{8i} N Z_c^{-(N+1)} \tag{6}$$

The b_N term is similar to the a_N term which has been derived for the image doublet in the iron shield. We will look only at dipoles or quadrupoles with a circular iron shield with a radius of R. The center of the circular iron corresponds to Z = 0. This iron is infinitely permeable ($\mu = \infty$) so that the method of images can be correctly applied. One must determine the location of the image, Z_r, the strength of the image Γ_r, and the image doublet angle α_r. First lets look at the location of the image. The image angle Θ_{cr} is the same as the doublet angle Θ_c (see Figure 1 and Figure 2). The radius of the image $r_r = R^2/r_c$ where r_c is the doublet radius. The doublet strength factor and doublet angle for the image can be found using the following form;

$$\Gamma_r = \left(\frac{R}{r_c}\right)^2 \Gamma \tag{7a}$$

$$\alpha_r = \pi + 2\,\Theta_c - \alpha \tag{7b}$$

See Figure 1 for an illustration. The corresponding terms for the piece of oriented material are;

$$M_r = \left(\frac{R}{r_c}\right)^2 M \tag{8a}$$

$$\Phi_r = \pi + 2\,\Theta_c - \Phi \tag{8b}$$

Using the values for Γ_r and α_r given in Eq. 7a and 7b one can come up with the b_N coefficient for Equation 4a using the current doublet

$$b_N = - \frac{\Gamma_e{}^{i(\pi - \alpha)}}{2\pi i} N \frac{(Z_c{}^*)^{N-1}}{R^{2N}} \tag{9}$$

where $Z_c{}^*$ is the complex conjugate of Z_c and R is the iron radius. The corresponding equation for the piece of oriented rare earth cobalt

$$b_N = - \frac{D^2 M\, e^{i(\frac{\pi}{2} - \Phi)}}{8i} N \frac{(Z_c{}^*)^{N-1}}{R^{2N}} \tag{10}$$

Oriented Magnetized Material

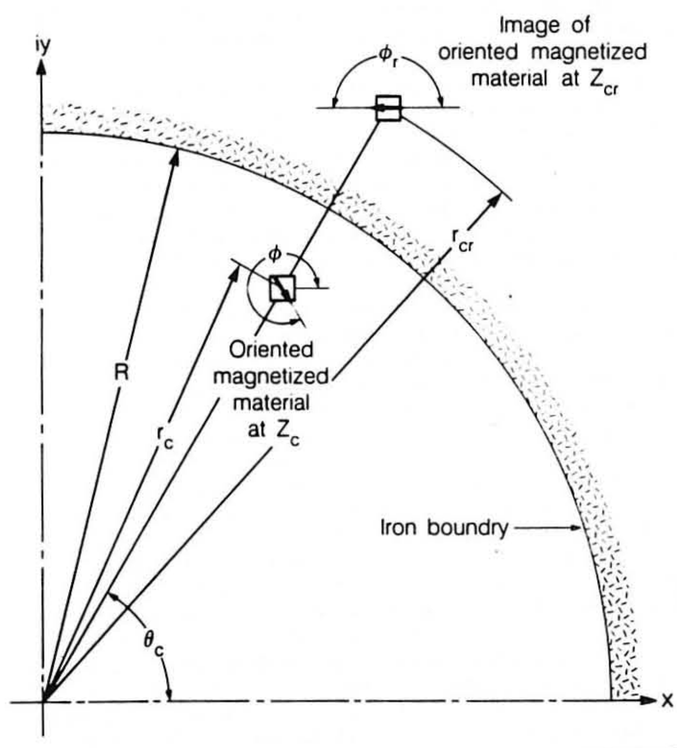

Fig. 2. Oriented magnetized material and its image in the complex plane.

the SCMAG04 program actually uses Equations 5 and 9 to expand the field. The doublet strength factor Γ and the doublet angle α are calculated from a given value of M and Φ which are properties of the oriented rare-earth cobalt material.

SOME MULTIPOLE MAGNET DESIGNS

The theory presented in this report can be used on a variety of rare earth cobalt permanent magnets with and without out iron. As an illustration of the theory, two kinds of two dimensional dipole and higher multipole magnets are discussed here. They are: the segmented type of dipole on higher multipole magnets described by Halbach [11,13,15] and a type described as a regular type where preshaped piece of material are oriented in the type of magnet which one is trying to create.

a) The segmented magnets

The segmented type of multipole magnet developed by Halbach [13,15] has the property that the orientation angle Φ of the magnetization in the block has the following relationship to the location angle of the block θ_i;

$$\Phi = \frac{m\pi}{2} + (T + 1)\ \theta_c \tag{11}$$

Where T is the fundamental multipole of the magnet you want to build (T = 1) is dipole; T = 2 is quadrupole and so on). θ_c is the angular location of the block and Φ is the orientation angle of the magnetization in the material. If m is even (m = 0,2,3,4,6...) the magnet is a skew magnet of 2T poles. If m is odd (m = 1,3,5,....), the magnet is a normal magnet of 2T poles.

If the orientation angle varies continuously with θ as Equation 11 indicates, a perfect 2T pole magnet will result with no higher multipole terms. The images in the iron do not contribute to the field (all the b_N terms are zero). The field is the same whether the iron is present or not (the only role of the iron is shielding).

When one builds a segmented 2T pole magnet one uses a finite number of geometrically identical blocks. The number of these blocks will determine how good the magnet is the next highest multipole produced N = mM + T where m = 0,1,2,3 and so on. So an eight block M = 8 dipole T = 1 will have an N = 1 term. The next term will be N = 9. The next is N = 17 and so on. The iron shield contributes no fundamental term, but it does contribute N = mM - T. We find that the eight block M = 8 iron shielded dipole T = 1 has N = 7,9,15,17 and so on. The fundamental is the same in both cases.

The segmented multipole magnet utilizes the magnetic material fully. Iron does not change the fundamental field but it can change higher multipoles. A good field can be obtained and it is possible to achieve fields in the magnet which are higher than the magnetization induction in the rare earth cobalt material. Figure 3 shows a segmented rare earth cobalt twelve piece quadrupole which generates a good quality field. The major disadvantages of the segmented multipole magnets are; that several types of blocks have to be made (this is worse for a dipole than for a quadrupole) and the field quality is dependent on the accuracy of magnetization of the segments, the accuracy of the shape of the segments, and the accuracy of the placement of the segments.

b) The regular magnets

The regular magnet configuration is one where a ring of the powdered material is put into dipole or quadrupole field before the material is bonded. (The center of the ring corresponds to the axis of the quadrupole field when a quadrupole is made this way. Dipole magnets have no specific axis). The magnetized material has an orientation angle Φ which takes the following form as a function of the location angle θ_c;

$$\Phi = \frac{m\pi}{2} + (T - 1)\ \theta_c \tag{12}$$

where T is the fundamental multipole. θ_c is the angular location of the point in the ring; and Φ is the orientation angle of the material along a line at position angle θ_c. If m is even, the magnet is a skew magnet. If m is odd the magnet is a normal magnet.

The advantage of this type of magnet is the simplicity of manufacture. A dipole built this way is much easier to build than a segmented dipole magnet because the ring is a single piece (see Figure 4 for an example of the regular dipole magnet). The regular dipole ring of oriented rare earth cobalt does not generate any magnetic field inside the ring unless there is an iron shell around the ring. The sign of the field generated is opposite that of the orientation direction of the lines in the REC ring (the image doublets in the iron have the same Φ notation with respect to θ_c plus π as the doublet in the segmented type of magnet. (See Equation 8b). A regular dipole with produce a perfect dipole field as long as the ring axis corresponds to the axis of the circular shield. The orientation of the magnetic particles must not be disturbed during the bonding process.

The disadvantage of the regular type magnet is that the oriented rare earth cobalt magnetic material is not fully utilized. The penalty may be acceptable in a dipole magnet because the construction is greatly simplified, but it is not generally acceptable in a quadrupole or sextupole magnet. Table 1 shows the fundamental multiple at 1 cm radius for both the regular type and a twelve segment type of dipole (T = 1), quadrupole (T = 2) and sextupole (T = 3). These magnets have oriented magnetized rare earth cobalt material with μ_0 M = 1.1R. This material is located between R = 2 cm and R = 4 cm as shown in Figures 3 and 4. The iron shield is at R = 4 cm. Table 1 shows that the regular type multipole magnet will only produce field when the iron is present.

Table 1.

Fundamental Multipole at a 1 cm Radius

Magnet Multipole Number	REGULAR TYPE w/o Iron	REGULAR TYPE w Iron	SEGMENTED TYPE w/o Iron	SEGMENTED TYPE w Iron
T = 1	0	0.412 T	0.730 T	0.730 T
T = 2	0	0.160 T	0.495 T	0.495 T
T = 3	0	0.048 T	0.256 T	0.256 T

Inner Material Radius = 2 cm

Inner Iron Radius = 4 cm

ORIENTED RARE EARTH COBALT
SEGMENTED QUADRUPOLE MAGNET
WITH CIRCULAR IRON

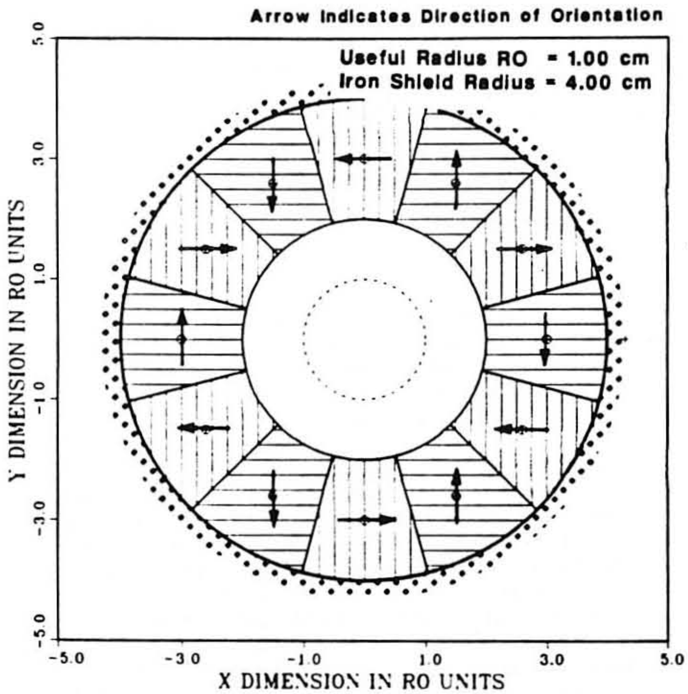

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Fig. 3.

ORIENTED RARE EARTH COBALT
NATURAL DIPOLE MAGNET
WITH CIRCULAR IRON

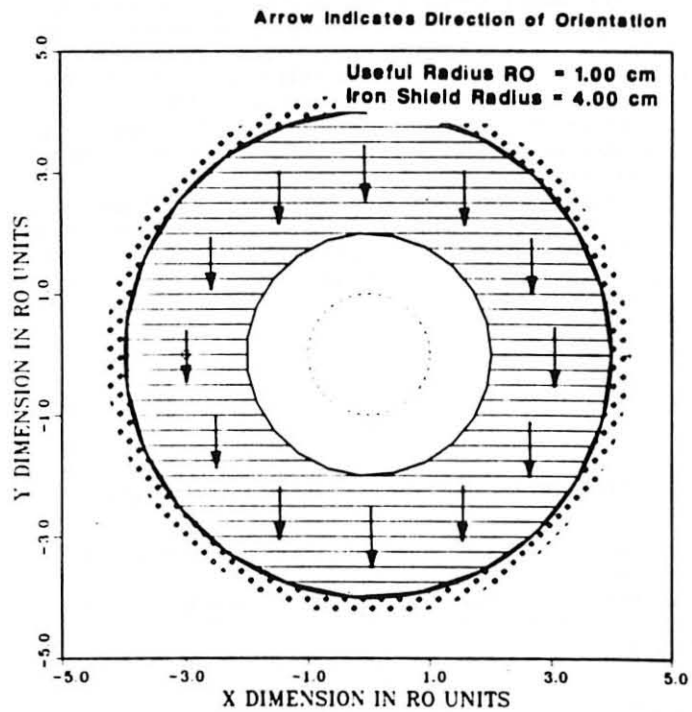

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Fig. 4.

GENERAL COMMENTS

The examples given here are trivial ones which can be calculated analytically [13]. The computer code can be used for magnets with complex shapes (particularly without iron). The permanent magnet material can be used in combination with current carrying blocks, small pieces of ferromagnetic material, and magnetized pieces of superconductor [7]. For a variety of cases one can simulate magnets with oriented rare earth cobalt materials without having to use finite element programs.

ACKNOWLEDGEMENTS

The author acknowledges the discussions he has had with Klaus Halbach of LBL over the last several years. This work was supported by the Office of Basic Energy Sciences Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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